

w -singularities in cosmological models

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Abstract. Recently a new type of cosmological singularity has been postulated for infinite barotropic index w in the equation of state $p = wp$ of the cosmological fluid, but vanishing pressure and density at the singular event. Apparently the barotropic index w would be the only physical quantity to blow up at the singularity. In this talk we would like to discuss the strength of such singularities and compare them with other types. We show that they are weak singularities.

1. Introduction

Until the observation of accelerated expansion of our universe, the only types of singularities that were considered to appear in Friedman-Lemaître-Robinson-Walker (FLRW) cosmological models were singularities in the form of a big bang or a big crunch, for which the density of the matter content of the spacetime blows up [1], as well as the scale factor of the universe. However, since the observational value of the barotropic index of the universe nowadays (the quotient between the pressure and the density of the content of the universe, $w = p/\rho$) is around minus one, there is a possibility of violation of the energy conditions and new types of singularities could appear. In fact, such singularities appear naturally in gravitational theories which try to cope with the observed phenomena.

Many of these new singularities are included in a descriptive classification due to Nojiri, Odintsov and Tsujikawa (N.O.T. in the following), which takes into account which physical quantities (scale factor of the universe a , Hubble ratio H , pressure p or density ρ) are singular [2]:

- Big bang / crunch: Vanishing a , divergent H , ρ and p .
- Type I: “Big rip”: Infinite a , ρ and p [3].
- Type II: “Sudden”: Finite a , H and ρ , divergent \dot{H} and p [4].
- Type III: “Big freeze”: finite a , infinite H , ρ and p [5].
- Type IV: Finite a , H , \dot{H} , ρ and p , but infinite higher derivatives of a .

The strength of these singularities [6, 7] has been checked in [8, 9]. Summarizing, big rip singularities are strong whereas sudden [10], big freeze and type IV singularities are weak. In this sense, the latter cannot be taken as the final stage of the universe, since the spacetime could be extended continuously beyond the singularity.

Out of this scheme, directional singularities for which some observers experience an infinite curvature, though the curvature scalars vanish at the singularity, have been found in some theories [11].

Also out of this classification, in [12] a FLRW cosmology is shown in which only the barotropic index w becomes infinite at $t = t_s$,

$$a(t) = \frac{a_s}{1 - \frac{3\gamma}{2} \left(\frac{n-1}{n-\frac{2}{3\gamma}} \right)^{n-1}} + \frac{1 - \frac{2}{3\gamma}}{n - \frac{2}{3\gamma}} \frac{na_s}{1 - \frac{2}{3\gamma} \left(\frac{n-\frac{2}{3\gamma}}{n-1} \right)^{n-1}} \left(\frac{t}{t_s} \right)^{\frac{2}{3\gamma}} + \frac{a_s}{\frac{3\gamma}{2} \left(\frac{n-1}{n-\frac{2}{3\gamma}} \right)^{n-1} - 1} \left(1 - \frac{1 - \frac{2}{3\gamma}}{n - \frac{2}{3\gamma}} \frac{t}{t_s} \right)^n, \quad (1)$$

for $\gamma > 0$ and $n \neq 1$ a natural number, whereas the scale factor, $a(t_s) = a_s$, and all its derivatives remain finite and the density and the pressure vanish at t_s . In [13] similar singularities are obtained but with non-vanishing pressure.

This new type of singularities has been dubbed barotropic index w -singularities [12], or simply w -singularities, or even type V [14] singularities, remarking the difference between them and type IV singularities, since no higher derivatives of the scale factor blow up at the time of the singularity t_s .

In this talk we would like to show a characterization of barotropic index w -singularities in FLRW cosmologies in terms of the form of the scale factor. More details may be found in [15].

In the next section we derive a characterization of barotropic index w -singularities in terms of the exponents of the power expansion in time of the scale factor of the universe. A discussion of the results is included in the last section.

2. Characterization of barotropic index w -singularities

In gravitational theories such as general relativity, the content of the universe is depicted as a perfect fluid of density ρ and pressure p , depending on just the time coordinate t , due to the homogeneity and isotropy of FLRW spacetimes. This means that there is an equation of state for the perfect fluid, $p = p(\rho)$, and the ratio of pressure and density is defined as the barotropic index of the fluid, $w = p/\rho$. This index evolves with time except for power-law cosmologies, which have a constant barotropic index.

In flat FLRW cosmologies the barotropic index is written in terms of the scale factor of the universe and its derivatives,

$$\rho = 3 \left(\frac{\dot{a}}{a} \right)^2, \quad p = - \left(\frac{\dot{a}}{a} \right)^2 - \frac{2\ddot{a}}{a}. \quad (2)$$

$$w = -\frac{1}{3} - \frac{2}{3} \frac{a\ddot{a}}{\dot{a}^2}, \quad (3)$$

and if we assume that it may be expanded as a generalized power expansion of time around the time of the singularity [16, 8]

$$a(t) = c_0(t_s - t)^{\eta_0} + c_1(t_s - t)^{\eta_1} + \dots, \quad \eta_0 < \eta_1 < \dots, \quad c_0 > 0, \quad (4)$$

where η_0, η_1, \dots are real numbers, the barotropic index can be expanded accordingly,

$$w(t) = w_0(t_s - t)^{\zeta_0} + w_1(t_s - t)^{\zeta_1} + \dots,$$

Table 1. Singularities in barotropic index.

η_0	η_1	η_2	w_s
Nonzero	(η_0, ∞)	(η_1, ∞)	Finite
Zero	$(0, 1)$	(η_1, ∞)	Infinite
	1	$(1, 2)$	Infinite
	1	$[2, \infty)$	Finite
	$(1, \infty)$	$[\eta_1, \infty)$	Infinite

and the exponents and coefficients of this expansion can be written in terms of the former ones.

This gives rise to a hierarchy of cases and subcases depending on the values of η_0 and η_1 [15] that are consigned in table 1. We notice that infinite barotropic indices appear just for vanishing η_0 exponent, except for $\eta_1 = 1$ with $\eta_2 \geq 2$.

The requirement of finite derivatives of the scale factor implies that all exponents η_i must be natural numbers. Hence, we are left just with the subcase of infinite barotropic index with $\eta_0 = 0$ and $\eta_1 \geq 2$.

From (2) and (4) we learn that vanishing density requires $\eta_1 > 1$ and vanishing pressure requires $\eta_1 > 2$ [15].

Hence we see that an infinite barotropic index with finite derivatives of the scale factor of the universe requires a vanishing density at the time of the singularity but not a vanishing pressure.

This may be summarized in the following statement:

Theorem: A FLRW cosmological model has an infinite barotropic index at a finite time t_s with finite derivatives of the scale factor of the universe if and only if $a(t)$ can be expanded as a Taylor series at t_s with a null linear term,

$$a(t) = c_0 + \sum_{n=2}^{\infty} c_n (t_s - t)^n. \quad (5)$$

Furthermore if we require a vanishing pressure either (barotropic index w -singularity), the quadratic term c_2 is also to be zero.

3. Conclusions

We have shown a simple characterization of barotropic index w -singularities in FLRW cosmological models in terms of Taylor expansions of the scale factor of the universe. The necessary and sufficient condition is just the vanishing of the linear term of the expansion. And if we consider singularities with vanishing pressure, then both the linear and quadratic terms must be zero.

Most exotic singularities are weak in the sense that when we consider finite objects they are not disrupted by tidal forces on experiencing the singularity, since the spacetime can be extended continuously beyond the singular event [8].

In that reference, the strength of the singularities was checked in terms of the exponents of the expansion (4) of the scale factor of the universe. Though barotropic index w -singularities were not considered, the results obtained there are general enough to be applied to cosmological models with vanishing linear term. In fact, it was shown that singularities in cosmological models with $\eta_0 = 0$ and η_1 larger or equal than one were weak [17] with both Tipler [6] and Królak's [7] criteria.

Table 2. Strength of singularities in FLRW cosmological models.

η_0	η_1	η_2	$\{\eta_i\}$	Tipler	Królak	N.O.T.
$(-\infty, 0)$	(η_0, ∞)	(η_1, ∞)	I	Strong	Strong	I
0	$(0, 1)$	(η_1, ∞)	S	Weak	Strong	III
	1	$(1, 2)$	S	Weak	Weak	II
		$[2, \infty)$	S	Weak	Weak	IV
	$(1, 2)$	(η_1, ∞)	S	Weak	Weak	II
	$[2, \infty)$	(η_1, ∞)	S	Weak	Weak	IV
	2	$[3, \infty)$	N	Weak	Weak	w with $p_s \neq 0$
	$[3, \infty)$	$[\eta_1 + 1, \infty)$	N	Weak	Weak	w with $p_s = 0$
$(0, \infty)$	(η_0, ∞)	(η_1, ∞)	I	Strong	Strong	Crunch

Hence barotropic index w -singularities are weak and cannot be considered as a final stage for the universe, as it happens with other exotic singularities but big rip.

In table 2 we summarize our results on exotic singularities and their strength in terms of the exponents of the power expansion of the scale factor of the universe around the time of the singularity.

Singularities in the derivatives of the scale factor of the universe appear only for non-natural exponents in the expansion. This is the reason for including a column $\{\eta_i\}$ which deals with that fact.

The letter I (independent) means no additional condition on the exponents of the expansion around the time of the singularity. S (some) states that there must be at least one non-natural exponent in order to have a singularity in one of the derivatives. Finally, N (natural) means that all exponents are natural.

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